Detaljno ispitati date funkcije i nacrtati njihove grafike:

1. $f(x) = \dfrac{x}{x-2}$;

1) $x-2 \neq 0$
$x \neq 2$

$D = \mathbb{R} \setminus \{2\}$
$= (-\infty, 2) \cup (2, +\infty)$

2) $f(x) = 0$ $\quad x = 0$
$\dfrac{x}{x-2} = 0$ $\quad f(x) = 0$

$x = 0$

3) $f(x) > 0$

$\dfrac{x}{x-2} > 0$

	$(-\infty,0)$	0	$(0,2)$	2	$(2,+\infty)$
x	$-$	0	$+$	$\|$	$+$
$x-2$	$-$	$\|$	$-$	0	$+$
$f(x)$	$+$	0	$-$	$\|$	$+$

$f(x) > 0$ za $x \in (-\infty, 0) \cup (2, +\infty)$
$f(x) < 0$ za $x \in (0, 2)$

4) „N1-N1" je domen nije simetrican u odnosu na $(0,0)$

5) V.A.

$\lim\limits_{x \to 2^-} \dfrac{x}{x-2} = \dfrac{2}{0^-} = -\infty$

$\lim\limits_{x \to 2^+} \dfrac{x}{x-2} = \dfrac{2}{0^+} = +\infty$

$x = 2$ je V.A.

H.A. $\lim\limits_{x \to \pm\infty} \dfrac{x}{x-2} = 1$

$y = 1$ je H.A.

K.A. nema jer ima H.A.

6) $y' = \dfrac{x-2-x}{(x-2)^2}$

$y' = \dfrac{-2}{(x-2)^2}$, $x \neq 2$

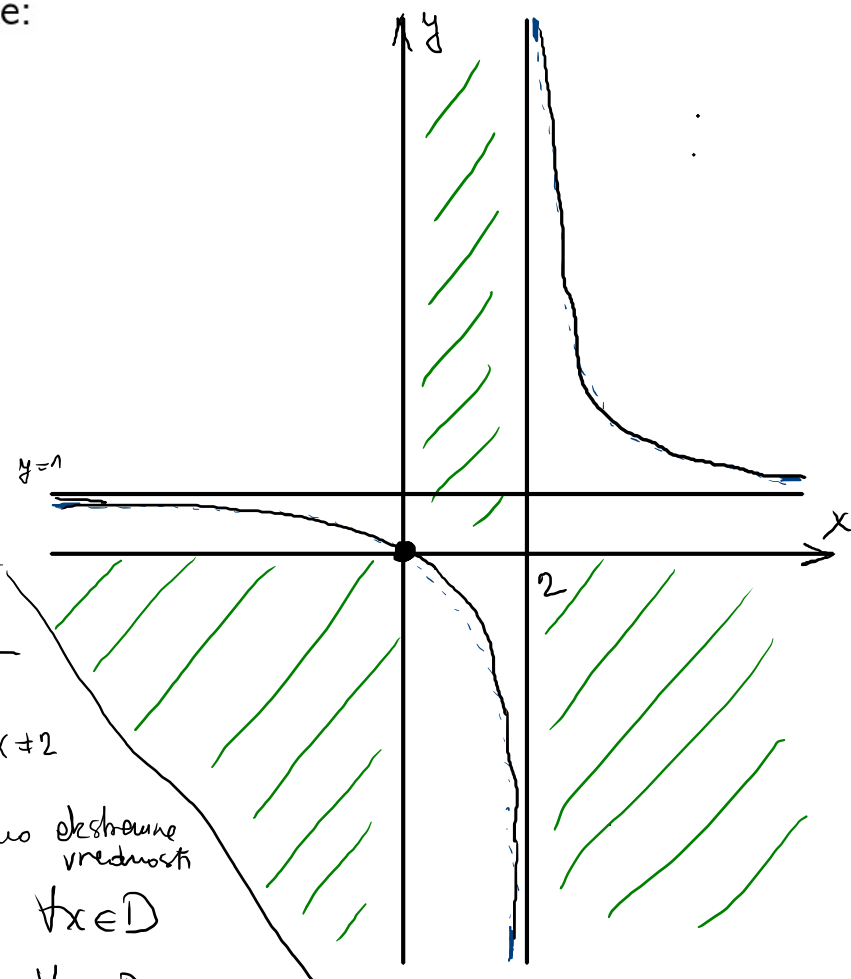
$y' \neq 0$ $\forall x \in D \Rightarrow$ nema ekstremne vrednosti

$y' > 0$

$-\dfrac{2}{(x-2)^2} < 0$

$y' < 0$ $\forall x \in D$

$y \searrow$ $\forall x \in D$



f,

$$y' = \frac{-2}{(x-2)^2}$$

$$y'' = \frac{-(-2)\,2(x-2)}{(x-2)^4} = \frac{4}{(x-2)^3}, \quad x \neq 2$$

$y'' \neq 0 \quad \forall x \in D \Rightarrow$ nema prevojnih točaka

$y'' > 0$

$$\frac{4}{(x-2)^3} > 0$$

$$(x-2)^3 > 0$$

$$x - 2 > 0$$

$$x > 2$$

$y'' > 0$ za $x \in (2, +\infty)$ $y \cup$

$y'' < 0$ za $x \in (-\infty, 2)$ $y \cap$

2. $f(x) = \dfrac{x^2}{x-2}$;

1) $x-2 \neq 0$

$x \neq 2$

$D = \mathbb{R} \setminus \{2\} = (-\infty, 2) \cup (2, +\infty)$

2) $y = 0$ za $x = 0$

3) $y > 0$

$\dfrac{x^2}{x-2} > 0$ ($x^2 \geq 0$)

$x - 2 > 0$

$x > 2$

$y > 0$ za $x \in (2, +\infty)$

$y < 0$ za $x \in (-\infty, 2)$

4) „NI-NI"

5) V.A. $x = 2$ je V.A.

$\lim\limits_{x \to 2^-} \dfrac{x^2}{x-2} = \dfrac{4}{"0^-"} = -\infty$

$\lim\limits_{x \to 2^+} \dfrac{x^2}{x-2} = \dfrac{4}{"0^+"} = +\infty$

H.A.

$\lim\limits_{x \to \pm\infty} \dfrac{x^2}{x-2} = \pm\infty$

Nema H.A.

K.A. $y = kx + n$

$k = \lim\limits_{x \to \pm\infty} \dfrac{f(x)}{x} = \lim\limits_{x \to \pm\infty} \dfrac{\frac{x^2}{x-2}}{x} = \dfrac{x^2}{x-2} \cdot \dfrac{1}{x}$

$= \lim\limits_{x \to \pm\infty} \dfrac{x}{x-2} = 1$

$n = \lim\limits_{x \to \pm\infty} (f(x) - kx)$

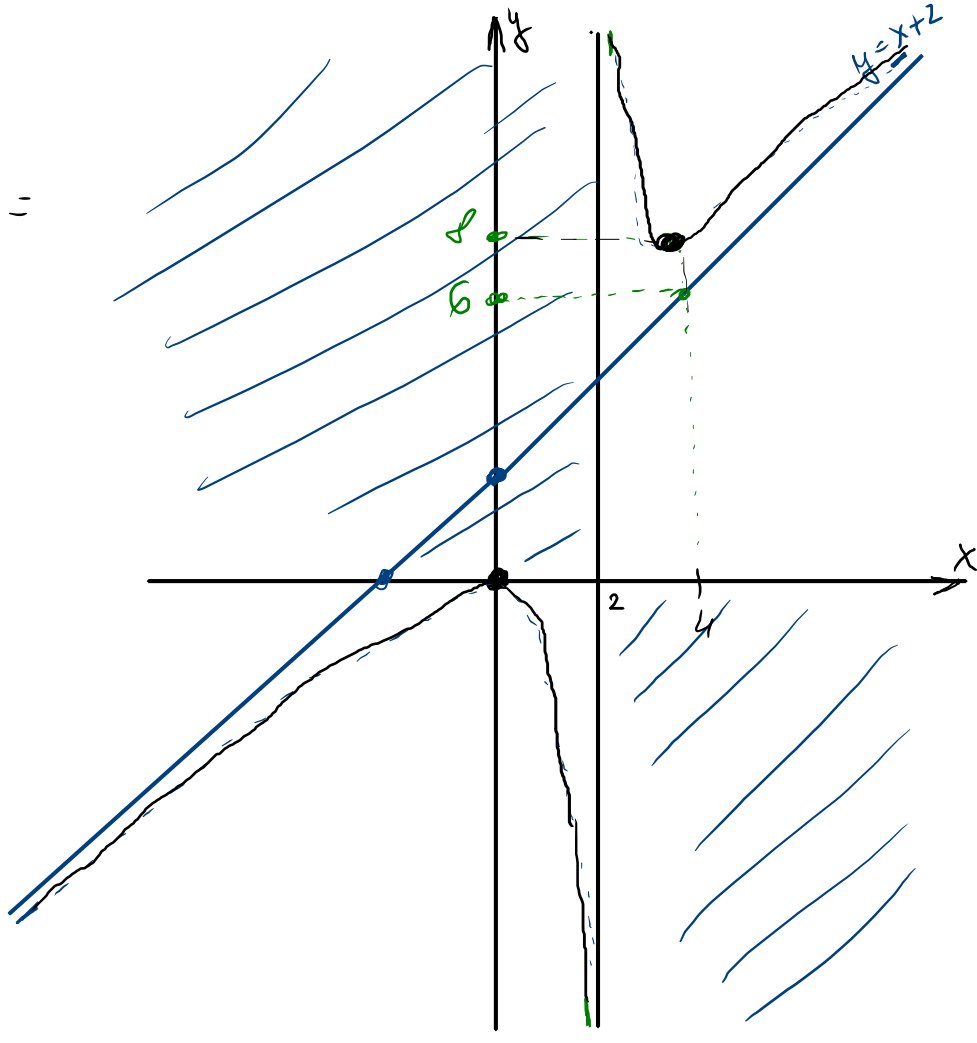
$= \lim\limits_{x \to \pm\infty} \left(\dfrac{x^2}{x-2} - x\right)$

$\infty - \infty$ $= \lim\limits_{x \to \pm\infty} \dfrac{x^2 - x^2 + 2x}{x-2} = 2$

$y = x + 2$ je K.A.

$x = 0 \quad y = 2$

$y = 0 \quad x = -2$



$$y = \frac{x^2}{x-2}$$

$$D = \mathbb{R}\setminus\{2\} = (-\infty, 2) \cup (2, +\infty)$$

6, $$y' = \frac{2x(x-2) - x^2}{(x-2)^2} = \frac{2x^2 - 4x - x^2}{(x-2)^2} = \frac{x^2 - 4x}{(x-2)^2} = \frac{x(x-4)}{(x-2)^2}$$ $x \neq 2$

$y' = 0$

$$\frac{x(x-4)}{(x-2)^2} = 0$$

$$x(x-4) = 0$$

$\boxed{x=0}$ $\boxed{x=4}$

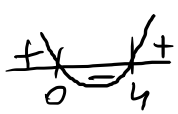
TAČKE U KOJIMA MOŽE A NE MORA BITI EKSTREMNA VREDNOST

$y' > 0$

$$\frac{x(x-4)}{(x-2)^2} > 0, \quad (x-2)^2 > 0$$

$$\boxed{x(x-4) > 0}$$

I način

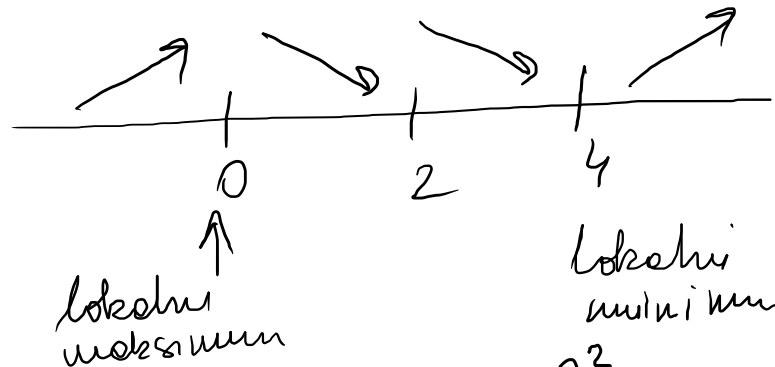


II način

		0		4	
x	−	0	+		+
$x-4$	−		−	0	+
y'	+		−		+

$y' > 0$ ZA $x \in (-\infty, 0) \cup (4, +\infty)$ $y \nearrow$

$y' < 0$ ZA $x \in (0, 2) \cup (2, 4)$ $y \searrow$



$x = 0$ $y_{max}(0) = \frac{0^2}{0-2} = 0$

$x = 4$ $y_{min}(4) = \frac{4^2}{4-2} = \frac{16}{2} = 8$

$$y' = \frac{x(x-4)}{(x-2)^2} = \frac{x^2-4x}{(x-2)^2}$$

7,
$$y'' = \frac{(2x-4)(x-2)^2 - (x^2-4x)\,2(x-2)}{(x-2)^4}$$

$$= \frac{2(x-2)\left[x^2-4x+4-x^2+4x\right]}{(x-2)^{4-3}} = \frac{8}{(x-2)^3}, \quad x \neq 2$$

$y'' \neq 0 \quad \forall x \in D \Rightarrow$ nemo prevojne tačke

$y'' > 0$

$$\frac{8}{(x-2)^3} > 0$$

$$(x-2)^3 > 0$$

$$x-2 > 0$$

$$x > 2$$

$y'' > 0 \quad x \in (2, +\infty)$ y $\cup$

$y'' < 0 \quad x \in (-\infty, 2)$ y $\cap$