(1) $\sum_{n=1}^{\infty} \frac{n}{(2n+2)\,5^n}$ $\qquad a_n = \frac{n}{(2n+2)\,5^n}$

$$\lim_{n\to\infty} \frac{a_{n+1}}{a_n} = \lim_{n\to\infty} \left[\frac{\frac{n+1}{(2(n+1)+2)\cdot 5^{n+1}}}{\frac{n}{(2n+2)\cdot 5^n}} \right]$$

$$= \lim_{n\to\infty} \frac{(n+1)\cdot 5^{n}\cdot (2n+2)}{n\cdot(2n+4)\cdot 5^{n+1}} = \lim_{n\to\infty} \frac{2n^2+4n+2}{10n^2+20n} = \frac{2}{10} = \frac{1}{5} < 1$$

(2) $\sum_{n=1}^{\infty} \frac{n+3}{\sqrt[3]{n^2+5}}$ $\qquad a_n = \frac{n+3}{\sqrt[3]{n^2+5}} \sim \frac{n}{\sqrt[3]{n^2}} = \frac{n}{n^{2/3}}$ (K)

$\sum \frac{1}{n^{-\frac{1}{3}}}$ D $\qquad = \frac{1}{n^{-\frac{1}{3}}}$ $\qquad -\frac{1}{3} < 1$

$\Rightarrow \sum \frac{n+3}{\sqrt[3]{n^2+5}}$ D

(3) $\sum_{n=1}^{\infty} \frac{3^n}{(n+1)7^n}$

(1/2) $\frac{3}{7} \le \frac{3}{7}$ (1/3) $\frac{9}{49} \le \frac{9}{49}$ (1/4) $\left(\frac{3}{7}\right)^3 \le \left(\frac{3}{7}\right)^3$

$$a_n = \frac{3^n}{(n+1)7^n} = \frac{1}{n+1} \cdot \left(\frac{3}{7}\right)^n \le \left(\frac{3}{7}\right)^n$$

$\sum \frac{1}{n^\alpha}$ < K $\alpha > 1$; D $\alpha \le 1$

$\sum q^n$ < K $q < 1$; D $q > 1$

$\sum \left(\frac{3}{7}\right)^n$ (K) $\Rightarrow \sum \frac{3^n}{(n+1)7^n}$ (K)

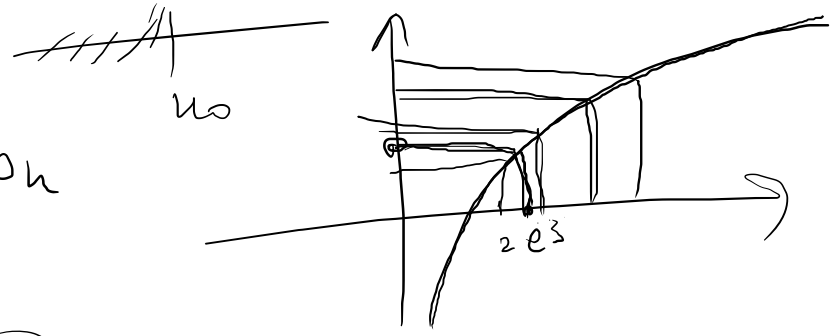
$q = \frac{3}{7} < 1$

(8)

$$\sum_{n=2}^{\infty} \frac{\ln n}{n}$$

$n \geq n_0$

$a_n \leq b_n$



$n \geq 3$

$$a_n = \frac{\ln n}{n} \geq \frac{\ln e}{n} = \frac{1}{n}$$

$$\sum \frac{1}{n} \; D \Rightarrow \sum \frac{\ln n}{n}$$

$n \geq 3$

$\ln n \geq \ln e$

$\ln n$

$1 = \ln e = \ln(2{,}73\ldots)$

(5) $\sum_{n=1}^{\infty} \frac{7^n}{(n+5)\,9^n}$

$a_n = \frac{1}{n+5} \cdot \left(\frac{7}{9}\right)^n \le \left(\frac{7}{9}\right)^n$

$\sum \left(\frac{7}{9}\right)^n \Longrightarrow \sum \frac{7^n}{(n+5)9^n}$ (K)

$q = \frac{7}{9} < 1$ (K)

$a_n \le b_n$

$\sum a_n \quad \sum b_n$

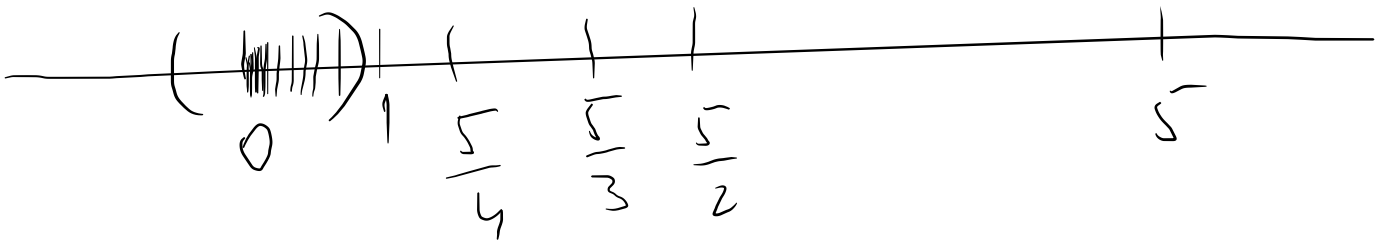
K $\Longleftarrow$ K

D $\Longrightarrow$ D

(6)

$$a_n = \frac{5}{n}$$

$$5, \frac{5}{2}, \frac{5}{3}, \frac{5}{4}, \ldots$$



(7)

$$\lim_{n\to\infty} \frac{1-\sqrt{n}}{2+3\sqrt[3]{n}} = \lim_{n\to\infty} \frac{\sqrt{n}\left(\frac{1}{\sqrt{n}}\to 0 - 1\right)}{\sqrt{n}\left(\frac{2}{\sqrt{n}}\to 0 + 3\frac{1}{\sqrt[3]{n}}\to 0\right)} = -\infty$$

$$\frac{\sqrt[3]{n}}{\sqrt{n}} = \frac{n^{\frac{1}{3}}}{n^{1/2}} = n^{\frac{1}{3}-\frac{1}{2}} = n^{\frac{2-3}{6}} = n^{-\frac{1}{6}} = \frac{1}{n^{\frac{1}{6}}} = \frac{1}{\sqrt[6]{n}}$$

(8)

$$\lim_{n\to\infty} \left(\sqrt{n+3} - \sqrt{n+7}\right) \cdot \frac{\sqrt{n-3} + \sqrt{n+7}}{\sqrt{n-3} + \sqrt{n+7}} =$$

$$= \lim_{n\to\infty} \frac{n-3-(n+7)}{\sqrt{n-3}+\sqrt{n+7}} = \frac{-10}{\infty} = 0$$

(9)

$f(x) = e^{2x}$, $A(0, y_0)$

$y_0 = f(0) = e^0 = 1$ $\quad A(0,1)$

$f'(x) = 2e^{2x}$

$f'(0) = 2e^0 = 2$

$t: \; y - y_0 = f'(0)(x - x_0)$

$y - 1 = 2(x - 0)$

$y = 2x + 1$

$n: \; y - y_0 = -\frac{1}{f'(0)}(x - x_0)$

$y - 1 = -\frac{1}{2}(x - 0)$

$y = -\frac{1}{2}x + 1$

(10) $y = \sqrt{x-7}$ $\quad y' = \frac{1}{2\sqrt{x-7}}$

$f''(x)$

$$y' = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x} = \lim_{\Delta x \to 0} \frac{\sqrt{x+\Delta x-7} - \sqrt{x-7}}{\Delta x} \cdot$$

$$\cdot \frac{\sqrt{x+\Delta x-7} + \sqrt{x-7}}{\sqrt{x+\Delta x-7} + \sqrt{x-7}} = \lim_{\Delta x \to 0} \frac{x+\Delta x-7-x+7}{\Delta x\left(\sqrt{x+\Delta x-7} + \sqrt{x-7}\right)}$$

$$= \lim_{\Delta x \to 0} \frac{\Delta x}{\Delta x\left(\sqrt{x+\Delta x-7} + \sqrt{x-7}\right)} = \frac{1}{\sqrt{x-7}+\sqrt{x-7}} = \frac{1}{2\sqrt{x-7}}$$

(11)

$$\begin{cases} x = t^3 + 2\sin^2 t \\ y = \ln(t^2+1) \end{cases}$$

$$x'_t = 3t^2 + 2 \cdot 2 \sin t \cdot \cos t$$

$$y'_t = \frac{1}{t^2+1} \cdot 2t$$

$$y'_x = \frac{y'_t}{x'_t} = \frac{\frac{2t}{t^2+1}}{3t^2 + 4\sin t \cos t}$$

(12)

$$4xy + y^3 = 4$$

$x' = 1$

y'

$y = f(x)$

$$4(y + xy') + 3y^2 \cdot y' = 0$$

$\sin x$

(13)

$$y = (2+x)^{\sin x} \quad / \ln$$

$$\ln y = \ln (2+x)^{\sin x}$$

$$\ln y = \sin x \cdot \ln(2+x)$$

$$\frac{1}{y} y' = \cos x \cdot \ln(2+x) + \sin x \frac{1}{2+x}$$

$$y' = (2+x)^{\sin x} \left(\cos x \ln(2+x) + \frac{\sin x}{2+x} \right)$$